Terminal Location Planning in Intermodal Transportation with Bayesian Inference Method

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ABSTRACT

In this project, we consider the planning of terminal locations for intermodal transportation systems. For a given number of potential terminals and coexisted multiple service pairs, we find the set of appropriate terminals and their locations that provide the economically most efficient intermodal operation.

The first part of this project is to develop a two-layer Markov Chain Monte Carlo (MCMC)-based method to implement the terminal location planning. The lower-layer is an optimal routing algorithm for all service pairs that considers both efficiency and fairness for a given planning. The upper-layer is a planning algorithm based on MCMC with a stationary distribution mapped from the transportation cost function. This method has shown, as tested in various network scenario, better performance than a recently developed method using a greedy randomized adaptive search procedure together with a heuristic search procedure (GRASP-HEP).

In the second part of the project, we bring the probabilistic nature in transportation networks into consideration. Estimates of traffic needing to use the network, capacity of terminals and costs of using portions of the network vary time to time. Effects of these variations have not been previously studied in the literature. We characterize the uncertainty of the system parameters with probability density functions (PDFs) based on prior information, while map the cost function into a likelihood function. Then, the design problem can be converted into a Bayesian inference problem of finding parameter set solutions with high posteriori probability that is proportional to the product of the prior PDF and the likelihood. We have developed theoretic methods for uniform sampling multi-dimensional simplex volume and implemented the Nested Sampling method to rank solutions based on their evidence values.

This project has broader impact. Since the probabilistic features are inherent in transportation, the design model based on Bayesian inference with MCMC has the potential to provide a unified framework not only for the location planning but also for many other optimization problems in intermodal transportation systems. In addition, to enhance higher education, a graduate student in the PhD level has been recruited and educated through this project to focus on the probabilistic modeling and analysis of various problems in the intermodal transportation.
ACKNOWLEDGMENTS

We would like to thank Xiaobin Wu, a MS degree student in Department of Electrical Engineering at the University of Mississippi, for his assistance in developing some MCMC-based programs. We also appreciate Prof. Burak Eksioglu from the Mississippi State University and Prof. Waheed Uddin from The University of Mississippi for helpful discussion of the practical transportation issues throughout this project.
TABLE OF CONTENTS

ABSTRACT ..............................................................................................................i
ACKNOWLEDGMENTS .........................................................................................ii
TABLE OF CONTENTS ..........................................................................................iii
LIST OF TABLES .....................................................................................................iv
LIST OF FIGURES ...................................................................................................v
INTRODUCTION ......................................................................................................1
OBJECTIVE ...........................................................................................................2
SCOPE ...................................................................................................................3
METHODOLOGY .....................................................................................................5
  For Fixed Network Parameters ...........................................................................5
  For Varied Network Parameters ....................................................................6
DISCUSSION OF RESULTS ..................................................................................8
CONCLUSIONS ....................................................................................................11
RECOMMENDATIONS ........................................................................................12
ACRONYMS, ABBREVIATIONS, AND SYMBOLS .............................................13
REFERENCES .....................................................................................................14
LIST OF TABLES

TABLE 1. Data Structure of TABLE $M_{ij}$ for service pair $(i, j)$..........................5
TABLE 2. Planning and routing results for an I8K8 instance.........................8
TABLE 3. Comparison with GRASP-HEP......................................................9
TABLE 4. Terminal capacity and cost..........................................................10
TABLE 5. Total Cost and Evidence of each configuration.............................10
LIST OF FIGURES

Fig. 1. Intermodal transportation system.................................................................2
Fig. 2. Simplex volume (tetrahedron in 3-D).............................................................6
Fig. 3. A random network I8K8, and the service pairs ..............................................8
Fig. 4. Map illustrating service nodes (‘o’) and terminals (‘x’).................................10
INTRODUCTION

Intermodal transportation can significantly increase the economic competitiveness, by improving productivity and efficiency with reduced delay, congestion and operational cost compared to current transportation systems. According to [2][3], the transportation system is responsible for about 30% of the total greenhouse gas emission. Therefore, design of efficient intermodal transportation systems also helps to sustain a more amiable environment by significantly reducing the total CO2 emissions.

Considering the interconnection of the road mode and other modes such as railway transportation via terminals so that the road mode focuses more on many low-flow localized services and the railway mode acts as the major backbone for high capacity and large range services, one specific problem is to decide on the number of terminals and their locations given a set of potential terminals and to determine the route paths of difference services. The current research in this area has considered different intermodal representation models [3]–[5] and different heuristic optimization methods [6]–[9]. In [3], an overview of the most prominent research efforts within operational research in the intermodal transportation has been provided. Compared with simulation-based techniques [4], using network models becomes more popular in research. In [5] an overview is provided on several network models based on which the optimization process is then carried out. As proven in [9], this terminal location planning problem is an NP-hard problem and hence the deterministic methods are impractical when the size of the network grows. As a result, when the number of nodes increases, an optimization method based on the heuristic search are applied, such as using simulated annealing [6], genetic algorithms [7], Tabu search [8] and greedy randomized adaptive search procedure and attribute-based hill climber [9].

Despite research progress in this area, there are two prominent issues that have either not been completely solved or not studied. First, one key issue in heuristic methods is to find the candidate solutions iteratively. Solutions generally are generated randomly and then passed for some criteria tests to decide whether they should be discarded or put into a pool. This process is often very time consuming and is done case-by-case in different heuristic methods. It is more desired to develop a unified technique that can guide this “random walk” in the multi-dimensional decision space so that candidates are generated more efficiently based on their potential contribution to the final solution. Second, all current work assumes that the parameters in the transportation network are stationary. In fact, these parameters can change significantly either continuously or from time to time. For example, the loads of demand from one city to another may be different in different time. In addition, the capacity of roads and the terminals can also change due to construction and maintenance. Therefore, the parameter variation is an inherent probabilistic feature in a transportation network and should be considered in the terminal planning. This issue, not being considered in the literature yet, is important particularly because the terminal planning, once being decided, is associated with a long-term operation.
OBJECTIVE

Fig. 1. Intermodal transportation system

We consider an intermodal transportation system consisting of two modes as shown in Fig. 1 that are interconnected with terminals. For simplicity, they can be perceived as road and railway transportation modes, respectively. In such a system, services are offered between a certain number of origin-destination locations, such as represented by service nodes 1 to 9. In general, the direct delivery service using a specific carrier is very expensive and the number of direct deliveries is often limited. In order to improve the economic competitiveness, low-volume demands can be moved to a consolidation terminal, such as represented by nodes A, B, C, via the road mode. In these terminals, the large number of low-volume freight will be consolidated into high-volume flows that will be routed to other terminals through high-frequency, high-capacity services that could be operated in the railway mode. The large number of lower frequency services, often operated with smaller vehicles, are used between the terminals and the origin/destination nodes. Without the loss of generality, we also allow the customized direct service using paths between an origin node and a destination node that does not go through any consolidation terminals.

The objective of this project is to solve the intermodal terminal location planning problem through Bayesian inference method with Markov Chain Monte Carlo (MCMC). There are two technical goals in this project. One is to design a MCMC-based method that can solve the terminal location planning problem when the network parameters are fixed as assumed in all existing work. The other is to design the Bayesian inference method that brings the inherent probabilities features in transportation network into the consideration of terminal planning. In addition, the project also has an education objective that recruits and involves a PhD student into the research in intermodal transportation systems with focus on optimal system design.
SCOPE

To illustrate the research problem, we adopt the basic model proposed in [9] and describe the transportation with a graph network. Let I be the set of all origin-destination service pairs and K the set of all potential terminal locations in the network. Each origin-destination pair \((i,j)\) has associated with it a positive and fixed amount \(q_{ij}\) of goods that need to be transported \((q_{ii}=0)\). The variable \(x_{ij}\) represents the fraction of the demand \(q_{ij}\) transported uni-modally, whereas the set of variables \(x_{ij}^{gk}\) relate to the fraction of the demand \(q_{ij}\) transported intermodally using terminals \(g,k \in K\). Let \(c_{ij}^{gk}\) be the unit cost of transporting demand between \(i\) and \(j\) through terminals \(g\) and \(k\) and \(c_{ij}\) be the unit cost of transporting demand directly from \(i\) to \(j\) without any intermediate intermodal operations. \(c_{ij}^{gk}\) is generally much less than \(c_{ij}\) due to the encouragement of intermodal transportation and the high cost of direct transportation. For each potential terminal location \(k \in K\), it has been associated with a positive and fixed capacity \(C_k\), a fixed cost \(F_k\) proportional to the capacity, and a decision variable \(y_k\) which is “1” when terminal \(k\) is open and “0” otherwise. Then the overall cost function is given as

\[
J = \sum_{i,j \in I} \sum_{g,m \in K} c_{ij}^{gm} x_{ij}^{gm} + \sum_{i,j \in I} c_{ij} w_{ij} + \sum_{k \in K} F_k y_k. \tag{1}
\]

This objective function represents the total transportation cost associated with all transportation flows within the network. Therefore, the terminal planning problem becomes to find the decision variables \(y_k, x_{ij}\) and \(x_{ij}^{gk}\) that minimize the function of \(J\), subject to the following constraints:

\[
x_{ij} \leq q_{ij} y_k, \quad \forall g,k \in K, \forall (i,j) \in I \tag{2}
\]

\[
x_{ij}^{gk} \leq q_{ij} y_k, \quad \forall g,k \in K, \forall (i,j) \in I \tag{3}
\]

\[
\sum_{g,k \in K} x_{ij}^{gk} + x_{ij} = q_{ij}, \quad \forall (i,j) \in I \tag{4}
\]

\[
\sum_{i,j : (i,j) \in I} \sum_{g \in K} x_{ij}^{g} + \sum_{i,j : (i,j) \in I} \sum_{g \in K} x_{ij}^{kg} \leq C_k, \quad \forall k \in K \tag{5}
\]

\[
x_{i,j} \geq 0, \quad x_{ij}^{gk} \geq 0, \quad x_{ij}^{kk} = 0, \quad \forall (i,j) \in I, \forall g,k \in K \tag{6}
\]

\[
y_k \in \{0,1\}, \quad \forall k \in K \tag{7}
\]

The objective function (1) represents the total transportation cost associated with all transportation flows within the network and consists of the sum of three terms. The first term represents the cost of flows through the intermodal transportation. The second term refers to the cost of the flows using the uni-modal transportation. The third term denotes the operation cost for all the terminals. Constraints (2) and (3) ensure that one flow can only go through those opened terminals. Constraint (4) shows that the sum of flows transported from
intermodal and uni-modal network must be equal to the overall demand associated with each origin-destination pair. Constraint (5) means that the overall flow going through a terminal cannot exceed the capacity of the terminal. Constraint (6) ensures that the amount of flow is non-negative and one flow cannot go through one terminal only. Constraint (7) means that one terminal is either used (open) or not used (not open).

Note that the above formulation also involves an underlying operation that simultaneously determines the optimal route paths for each service pair \((i,j)\), given a set of selected terminal locations. That is, \(x_{ij}^g\) and \(x_{ij}\) are in fact related to the route paths associated with \(y_k\) and other variables \(\Omega = \{q_{ij}, F_k, C_k\}, k \in K\). For given \(\Omega\), there are two issues.

- Depending on a particular terminal planning, i.e., the open/close status of potential terminals, determine the optimal routing paths for all service pairs simultaneously.
- Determine the set of the most appropriate terminals that minimizes the overall cost function.

The first task of the project is to design a MCMC-based method to implement the terminal location planning problem and compare with the state-of-the-art method.

The location problem is even more difficulty when \(\Omega\) varies. For the second task of this project, we solve the problem under varying \(\Omega\) through the Bayesian inference. We model the probabilistic feature with Gaussian probability density functions (PDF) as the prior information and calculate the Bayesian evidence of possible solutions. This work also includes the design of sampling methods uniformly in the prior.
**METHODOLOGY**

The mathematical technology used in this research is the Bayesian interference method with MCMC.

For Fixed Network Parameters

Define \( y = \{ y_k, k \in K \} \) as the selection of terminals; \( x = \{ x_{ij}^{gk}, (i,j) \in I; g,k \in K \} \) as the routing of service loads. Then \( D = \{ y, x \} \) is the decision variables, while \( \Omega \) is the network parameters. We first need to map the cost function into a probability function. This is represented as

\[
P_S(D|\Omega) = \frac{1}{Z_S} \exp \left[ -\frac{J(D|\Omega)}{S} \right]
\]

where \( S \) is a scalar used to adjust the shape of the probability function. \( Z_S \) is the normalization constant to ensure that the function is a probability distribution. \( J(D|\Omega) \) is the cost function for one set of decision variables \( D \), given system setup parameters \( \Omega \). Clearly a small routing cost gives a large probability value in this inference/optimization problem. The purpose is to find a specific decision instance \( D \) that maximizes \( P_S \). We solve the problem with a two-layer strategy.

- lower-layer: a table-based method that finds \( x \), given \( y \), i.e., finding the route paths for all services pairs simultaneously, given a particular terminal planning.
- upper-layer: a MCMC-based method that finds \( y \) that maximizes \( P_S \).

**Lower-layer algorithm**

For each service pair \((i,j)\), we first construct a routing information table \( M_{ij} \), and then based on all these tables we find the final routing result for each service pair and store the information in a table \( R_{ij} \). Table structure is shown in TABLE 1

<table>
<thead>
<tr>
<th>i</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>j</th>
<th>Unit Cost</th>
<th>Path Capacity</th>
</tr>
</thead>
</table>

Each row of this table stores one possible routing path for service \((i,j)\). \( T_1 \) and \( T_2 \) are intermediate terminals. When \( T_1 = T_2 = 0 \), the path corresponds to the uni-modal transportation. All tables are listed from the most efficient path to the least efficient path. For a particular path that is demanded by multiple service pairs, an urgency factor is defined as the ratio of the routing cost through current path and the cost through the next best path. This factor is utilized in a way that path of interest can be shared by multiple service pairs both
fairly and efficiently.

**Upper-layer algorithm**

This algorithm uses a MCMC method. Given the current candidate planning $y_1$, whether to accept next planning $y_2$ is based on the probability of

$$\min \left\{ 1, \exp \left( -\frac{J(y_2|\Omega) - J(y_1|\Omega)}{S} \right) \right\}.$$ 

This is termed as the Metropolis-Hastings process [10] that guarantees that samples drawn from MCMC converge to the PDF of $P_S$. To generate $y_2$ that has high accepting probability, slice sampling method presented in [11,12] is further employed.

**For Varied Network Parameters**

We capture the variation of network parameters $\Omega$ with model as a PDF $\Pi(\Omega)$. Gaussian distribution is used in this work. Therefore, our purpose now is to find a $D$ that maximizes

$$Z = \int \Pi(\Omega) P_S(D|\Omega) d\Omega.$$ 

$Z$ is also called the evidence of $D$ in Bayesian inference. This task is complete by mapping and using Nested Sampling method [13]. To start Nested Sampling, a key point is to generate the uniform samples in a multi-dimensional simplex volume.

![Simplex volume (tetrahedron in 3-D)](image)

As shown in Fig. 2, suppose there are three service pairs, each with traffic flow $x_1, x_2, x_3$ going through a specific terminal. Since the terminal capacity is fixed. With normalization, we have $x_1 + x_2 + x_3 \leq 1$. Nested sampling requires that we must be able to generate
(x_1, x_2, x_3) uniformly inside the simplex volume (i.e., the tetrahedron OABC in Fig. 2). The process becomes very complicated in a higher dimension when a large number of service pairs are involved. Through this project, we also solved this problem mathematically [14] by calculating a joint PDF for a function of multiple random variables. With this method, the Nested sampling can be further employed effectively to calculate the evidence of each possible location planning.
DISCUSSION OF RESULTS

The developed method and software (in MATLAB) has been tested in various network scenarios.

Considering Fixed Network Parameters

We have tested various random networks and compared with a recently developed method [9].

![Random network I8K8 and service pairs](image)

Fig. 3. A random network I8K8, and the service pairs

We denote “IaKb” as a random network that has “a” service nodes and “b” potential terminal locations. Terminal capacities are generated randomly and are associated with a proportional operation cost. Fig. 3 shows one instance for I8K8 and the service loads are also shown in the right of Fig. 3. The results by running our MCMC-based method for this example are shown in TABLE 2 where each row shows the routing information.

### TABLE 2. Planning and routing results for an I8K8 instance

<table>
<thead>
<tr>
<th>Origin (i)</th>
<th>Destination (j)</th>
<th>Units of Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>02</td>
<td>35</td>
</tr>
<tr>
<td>03</td>
<td>04</td>
<td>19</td>
</tr>
<tr>
<td>05</td>
<td>06</td>
<td>19</td>
</tr>
<tr>
<td>07</td>
<td>08</td>
<td>18</td>
</tr>
</tbody>
</table>

We have also compared the method with a recently developed state-of-the-art technique.
method: Greedy randomized adaptive search procedure-Heuristic evaluation procedure (GRASP-HEP) [9] in varies network setups. As shown in TABLE 3, our method has shown consistent improvement in various testing scenarios.

TABLE 3. Comparison with GRASP-HEP method

<table>
<thead>
<tr>
<th></th>
<th>I16K16</th>
<th>I20K12</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRASP-HEP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>19051</td>
<td>21649</td>
</tr>
<tr>
<td>Min</td>
<td>11946</td>
<td>16597</td>
</tr>
<tr>
<td>Avg</td>
<td>15591.4</td>
<td>18947.8</td>
</tr>
<tr>
<td>MCMC-Table Route</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>16327</td>
<td>20912</td>
</tr>
<tr>
<td>Min</td>
<td>10656</td>
<td>14767</td>
</tr>
<tr>
<td>Avg</td>
<td>13897.9</td>
<td>17326.2</td>
</tr>
</tbody>
</table>

| Avg Improvement  | 12.2 % | 9.4 % |

<table>
<thead>
<tr>
<th></th>
<th>I28K12</th>
<th>I30K20</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRASP-HEP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>30005</td>
<td>33597</td>
</tr>
<tr>
<td>Min</td>
<td>18298</td>
<td>22503</td>
</tr>
<tr>
<td>Avg</td>
<td>26348.1</td>
<td>28055.2</td>
</tr>
<tr>
<td>MCMC-Table Route</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>28267</td>
<td>32467</td>
</tr>
<tr>
<td>Min</td>
<td>17413</td>
<td>21248</td>
</tr>
<tr>
<td>Avg</td>
<td>24592.2</td>
<td>25749.1</td>
</tr>
</tbody>
</table>

| Avg Improvement  | 7.1 %  | 9.0 %  |

Considering Parameter Variations

The kind of historical intermodal transportation data we need is not freely available, so we instead invented a test case that attempts to be realistic. As shown in Fig. 4, we consider 14 cities and 4 existing rail terminals in five states (Mississippi, Alabama, Tennessee, Arkansas, and Georgia). The rail lines and terminals are all part of the existing Norfolk Southern intermodal freight network. The four terminals are Nashville, Memphis, Birmingham, and Atlanta) and arriving at each of the other cities in the network. The amount of freight demand coming from these cities is proportional to their populations. The proportion of the demand coming from one city going to another city is proportional to the destination city’s population as a percentage of the total population of all the cities in the network. Terminal capacities and fixed costs are proportional to the population of the nearest large city. The amount of demands may change and the variation is assumed to be Gaussian random variable with variance being one tenth of the mean value.

With the use of existing four terminals, three configurations were tested: no additional terminals added, one added near Chattanooga, and one added near Meridian. The costs and capacities of terminals are shown in TABLE 4. With our method, the total cost and the log-evidence of each configuration are found and shown in TABLE 5. The results show that the proposed Chattanooga terminal absorbs much of the demand that is normally routed uni-modally from Nashville, so the cost is noticeably lower. The proposed Meridian terminal, however, is not well placed and its utility to the network is minimal.
Fig. 4. Map illustrating service nodes (‘o’) and terminals (‘x’)

### TABLE 4. Terminal capacity and cost

<table>
<thead>
<tr>
<th>Terminal</th>
<th>Capacity (tons)</th>
<th>Fixed Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memphis</td>
<td>168,370</td>
<td>159,280</td>
</tr>
<tr>
<td>Huntsville</td>
<td>47,219</td>
<td>44,671</td>
</tr>
<tr>
<td>Birmingham</td>
<td>54,491</td>
<td>51,552</td>
</tr>
<tr>
<td>Atlanta</td>
<td>114,050</td>
<td>107,890</td>
</tr>
<tr>
<td>Chattanooga</td>
<td>96,349</td>
<td>96,800</td>
</tr>
<tr>
<td>Meridian</td>
<td>37,187</td>
<td>37,361</td>
</tr>
</tbody>
</table>

### TABLE 5. Total Cost and Evidence of each configuration

<table>
<thead>
<tr>
<th>Terminal</th>
<th>Cost ($)</th>
<th>Log-Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>None added</td>
<td>4,002,900</td>
<td>-4.0033e+10</td>
</tr>
<tr>
<td>Chattanooga</td>
<td>3,820,600</td>
<td>-3.8221e+10</td>
</tr>
<tr>
<td>Meridian</td>
<td>3,912,300</td>
<td>-3.9125e+10</td>
</tr>
</tbody>
</table>
CONCLUSIONS

Through this project, we have developed a MCMC-based method to solve the terminal location planning problem in intermodal transportation system. First, in the case without the consideration of network parameter variation as assumed in the literature, we have developed a two-layer MCMC-based method that outperforms one state-of-the-art method in various testing scenarios. Second, in the case where the parameter variation is considered, we model the variation with Gaussian priori probability distribution and study the terminal location problem based on their Bayesian evidence. This is a work not being found in the literature yet. Both mathematical results and software programs in MATLAB have been developed. In addition to this technical work, throughout this project, we have also involved a graduate student in the PhD level for the education purpose.

This project has also generated six oral presentations, one post presentation and three conference proceeding papers. One paper [14] presented in the 33rd International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering (MAXENT 2013) (Dec.15-20, Canberra, Australia) has won the Best Poster Award.
RECOMMENDATIONS

Through this project, we can conclude that the Bayesian inference method with MCMC could provide a unified methodology for solving various operational optimization problems in intermodal transportation systems. Compared with other case-by-case heuristic methods. This proposed work has an advantage to guide the “random walk” in the multi-dimensional decision space based on the samples’ contribution to a well-mapped stationary probability distribution. As a result, a better final solution can be approached, with a short time period. We highly recommend that this methodology can be fully appreciated by the research community in intermodal transportation systems.
# ACRONYMS, ABBREVIATIONS, AND SYMBOLS

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCMC</td>
<td>Markov Chain Monte Carlo</td>
</tr>
<tr>
<td>GRASP</td>
<td>Greedy Randomized Adaptive Search Procedure</td>
</tr>
<tr>
<td>HEP</td>
<td>Heuristic evaluation procedure</td>
</tr>
<tr>
<td>IaKb</td>
<td>“a” number of service nodes and “b” potential terminal nodes.</td>
</tr>
</tbody>
</table>
REFERENCES


